BASIC DYNAMIC PRINCIPLES OF RESPONSE OF LINEAR STRUCTURES TO EARTHQUAKE GROUND MOTIONS

by

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Abstract

The basic principles of dynamic seismic analysis of elastic structures are reviewed and the important parameters governing structural response are noted. The concept of the response spectrum is stressed; its application in the analysis of multi-story structures by modal superposition techniques is discussed and the role played by the spectrum in revealing earthquake ground motion characteristics is commented upon. The procedure for modal spectrum analysis of multi-story structures is summarized and illustrated by a simple example.

Introduction

The rational design and construction of an earthquake-resistant structure requires a knowledge of the lateral forces developed in the structure by the earthquake. The uncertain nature of future ground motions and the difficulty of precisely evaluating the appropriate physical properties of real structures limits our ability to provide an exact calculation of these forces. Nevertheless, sufficient progress has been made to allow us to predict the general response of structures to earthquake ground motions and so to ensure their survival with reasonable confidence.

An earthquake generates random motions of the ground in both the vertical and horizontal directions. If we assume that there is negligible interaction between the ground and the building, the structural foundations are excited with these same ground motions and inertia forces are induced in the building by virtue of its mass. These forces can be evaluated theoretically by contemplating the structure as an idealized damped system of many degrees of freedom, subjecting its base to the transient erratic motion of the earthquake, and determining its response. While amenable to mathematics, this represents a complex dynamic problem, the essential features of which will be examined below. The presentation is intended as an introduction to the subject of earth-

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quake engineering and is based largely on the publications of Housner (1)*, Hudson (2), Clough (3) and Blume, Newmark and Corning (4).

Earthquake Response of Single-Story Structures

The simplest dynamic model which will reveal the essential parameters relating structural response to earthquake disturbance is the single-degree-of-freedom system. Later it will be shown in very general terms that the behaviour of many complex multidegree-of-freedom structures can be represented by an appropriate superposition of a number of these simple systems.

Figure 1 is a schematic representation of the deformed shape assumed by an idealized one-story structure whose rigid foundation undergoes a displacement equal to the earthquake motion, x, of the ground. Due to the flexibility of the structure and the inertia of its mass, m, the columns deform thereby permitting the mass to displace relative to the ground by the amount $u = y - x$, where y is the displacement of the mass from its original position. These displacements are a function of time, t, and the structure is therefore excited into motion which is opposed by the shear stiffness, k, of the columns and the inherent friction of the structure, which is a form of energy dissipation and which is normally referred to as the damping. For a linear elastic system the shear or lateral force, V, exerted by the columns on the mass and on the ground may be expressed as

$$V = ku \tag{1}$$

The friction of a structure can be described satisfactorily by the condition of viscous or linear damping. Linear damping results in damping forces directly proportional and opposed to the velocity; the proportionality constant, c, is known as the damping coefficient. When motion is restricted to translation in one direction** only (as represent-

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* Numbers in parenthesis refer to References at end of paper.

** Systems which can vibrate in both horizontal directions respond to both horizontal components of ground motion and have a resultant horizontal time response which varies continuously in direction. However, structures are normally designed to withstand ground motion components assumed to act non-concurrently in the direction of each of the main axes of the structure. It is also customary to ignore the influence of vertical vibrations because of the reserve strength of the framing for vertical (gravity) loads.

ed by u), the differential equation governing the response of the flexible structure may be written as

$$c \frac{du}{dt} + k\,u = \frac{-md^2y}{dt^2} = \frac{-md^2(x+u)}{dt^2}$$

or $$m\ddot{u} + c\dot{u} + k\,u = -m\ddot{x} \qquad (2)$$

where the superscript dot notation indicates differentiations with respect to t. $\ddot{x}$ is the horizontal component acceleration of the base or ground which is identical with that recorded by a strong-motion accelerometer during an earthquake.

If, as is normally encountered in building structures, the damping is small ($n < 0.2$) the solution of equation (2) for the response u at any time t after starting from rest, is given by the Duhamel integral (5) as

$$u(t) = \frac{T}{2\pi} \int_0^t \ddot{x}(\tau)\, e^{-n(\frac{2\pi}{T})(t-\tau)} \sin \frac{2\pi}{T}(t-\tau)\, d\tau \qquad (3)$$

where T = natural undamped period of vibration of the structure

$= 2\pi\sqrt{\frac{m}{k}}$

n = fraction of critical damping $= \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$. The critical damping, c_c, is the minimum value of c resulting in a non-oscillating response.

It should be noted from equation (3) that the dynamic response u of the structure is dependent on <u>the character of the structure</u>, defined by its natural period of vibration (a function of its stiffness and weight) and its damping, and on <u>the character of the ground acceleration</u>.

The character of the ground acceleration is exhibited by the integral appearing equation (3) namely

$$S(t) = \int_0^t \ddot{x}(\tau)\, e^{-n(\frac{2\pi}{T})(t-\tau)} \sin \frac{2\pi}{T}(t-\tau)\, d\tau \qquad (4)$$

We denote the maximum value of this integral by the symbol S_v. That is,

$$S_v = \left[\int_0^t \ddot{x}(\tau)\, e^{-n(\frac{2\pi}{T})(t-\tau)} \sin \frac{2\pi}{T}(t-\tau)\, d\tau\right]_{max} \qquad (5)$$

If the displacement u(t), equation (3), is differentiated expressions for the velocity, $\dot{u}(t)$, and acceleration, $\ddot{u}(t)$, will be obtained. For small damping and earthquake ground motions it is found (6) that the maximum value of $\dot{u}(t)$ differs only slightly from S_v and further, that the maximum relative displacement, maximum relative velocity and maximum absolute acceleration are simply related by the following expressions:

$$u(t)_{max} = S_d = \text{max. relative displacement} = \frac{T}{2\pi} S_v \quad (6)$$

$$\dot{u}(t)_{max} = S_v = \text{max. relative velocity} = S_v \quad (7)$$

$$\ddot{y}(t)_{max} = S_a = \text{max. absolute acceleration} = \frac{2\pi}{T} S_v \quad (8)$$

S_d, S_v and S_a are referred to as the displacement, velocity and acceleration response spectrum respectively.

The Response Spectrum

For a particular ground acceleration input record, and for a particular n and T, the integral defining S(t) can be evaluted and the maximum value, S_v, observed. The plot of such maxima for a range of structural periods T yields a graph or influence line of maximum velocity response which is the velocity response spectrum. A family of such spectrum curves can be obtained corresponding to different n values. A schematic representation of the concept of response spectrum is shown in Fig. 2.

By virtue of the complex nature of the ground acceleration, the computation required for the evaluation of the response spectrum is extremely great and is best done by analogue or digital computers. The velocity spectrum for the N-S component of the El Centro, Calif., earthquake (7) of May 18, 1940 is presented in Fig. 3.

The response spectrum could also be plotted in terms of displacement or acceleration. In view of the relations expressed by equations (6), (7), and (8), it is possible to show the complete picture of structural response on a single, four-way log grid plot, as illustrated in Fig. 4. The maximum velocity, acceleration and displacement experienced by a single-degree-of-freedom structure when excited by the El Centro earthquake can be read directly from the graph. Similar curves can be plotted for other ground disturbances.

It should be noted in Figs. 3 and 4 that the zero damped spectra are characterized by abrupt oscillations indicating that the response is very sensitive to small period changes. The modifying influence of damping is seen to be very marked; small amounts of damping reduce the sensitivity to period changes and produce a large reduction in maximum response, particularly at the short-period end of the spectrum.

Application of Response Spectrum

The maximum seismic force developed in the single-degree-of-freedom system as a result of a given earthquake is directly obtainable from the velocity spectrum of that earthquake. Thus, when the spectrum is known the maximum base shear, V_B, transmitted into the structure from the ground, which is equal to the lateral seismic force developed in the columns, may be directly expressed through equation (1) as

$$V_B = k\,u_{max} = k\,\frac{T}{2\pi}\,S_v$$

or alternatively

$$V_B = \left(\frac{2\pi}{T}\,\frac{S_v}{g}\right) W = \left(\frac{S_a}{g}\right) W = CW \qquad (9)$$

where W denotes the weight of the structure and g is the acceleration of gravity. C, known as the seismic coefficient, is essentially the acceleration spectrum and represents a plot of the base shear coefficient as a function of the period. Although it is a direct indication of the seismic forces produced by an earthquake, the velocity response, which is generally favoured in scientific studies, seems to represent a more fundamental property of the earthquake.

Earthquake Design Spectra

Before the spectrum concept outlined above can be used in practice it is necessary to establish appropriate earthquake design spectra. Although the exact nature of future earthquake ground motions is not known, a study of the characteristics of existing spectra shows that there are basic similarities between them which permits certain idealizations in the prediction of general response spectra for simple elastic systems. Based on numerous strong-motion records of past disturbances, Housner (1) has established average response spectra which may be taken to represent the average properties of future ground motions. These are shown in Fig. 5 and 6 and correspond to large magnitude earthquakes at moderate epicentral distances (<45 miles). In practice, the ordinates of these curves must be multiplied by scale factors appropriate to the severity of the design earthquake being considered; for the 1940 El Centro earthquake this factor is 2.7. These average spectra yield the maximum response to be expected when the system is subjected to ground motions of the stated intensity.

The designer may use other methods of predicting structural response to future earthquakes, such as the idealized spectra (4) represented by the broken bounding lines forming the two polygons located in the upper part of Fig. 4. These lines are based on the fact that, for structures with 5 - 10% critical damping, existing spectral characteristics reveal spectral accelerations, velocities and displacements which are approximately 2, 1.5, and 1 times as great as the maximum ground acceleration, maximum ground velocity and maximum ground displacement respectively, of the particular design earthquake. For the El Centro 1940 earthquake, these ground motions are represented by the polygon made up of the three broken bounding lines in the lower part of Fig. 4. For structures with less than 2% critical damping, the numerical coefficients mentioned above are very nearly doubled in each case.

To complete our description of response spectrum analysis of single-degree-of-freedom systems it is desirable to examine the effect of size and distance of earthquake on the general appearance of spectrum curves. This is conveniently summarized in Fig. 7 which shows Housner's (1) smooth representation of the average undamped velocity spectra for different ground motions.

The reduction of ground motion intensity (and structural response) with distance is apparent by a comparison of curves A and B. This reduction is particularly accentuated in the short-period range of the spectrum and is due to the fact that the high frequency components of ground motion are attenuated more rapidly than the lower frequency components. As a result, tall flexible (high period) structures at relatively long distances from the centre of an earthquake will be more nearly attuned to the long-period range of ground motions than low rigid (short period) structures and may be expected to experience a relatively greater response and to suffer a proportionately greater degree of damage than low, stiff structures. Conversely, a nearby earthquake will concentrate its effects on, and tend to cause the most severe damage to, low rigid structures. Finally, it may be concluded from curve C that, for small nearby earthquakes, the low period components of ground motion are relatively exaggerated in the absence of major attenuation and low, stiff structures are thereby more adversely excited and are apt to suffer more serious damage than tall flexible buildings.

The development of the response spectrum concept and the establishment of spectrum curves for all available strong-motion earthquake records represents a major break-through in the seismic analysis of engineering structures. In the next section it will be shown that the maximum response of a mode of vibration of a multi-story structure can be determined from the response spectrum curves for single-degree-of-freedom systems.

Earthquake Response of Multi-Story Structures

The seismic analysis of a multi-story structure is more complex than the corresponding analysis described for the simple oscillator of Fig. 1. However, for linear structures with small damping, the application of normal mode theory (8) offers a simplified approach to the problem.

According to the normal mode concept, a multi-story structure can be represented by a number of equivalent one-degree-of-freedom systems. In general, for an n degree of freedom structure there are n of these equivalent systems; they are characterized by the n natural periods and associated normal mode shapes (configurations) in which the actual structure may vibrate. The distinguishing feature of a normal mode vibration is the fact that the ratios of displacements of all parts of the system are maintained constant with time. When transiently excited by the earthquake disturbance the equivalent single-degree-of-freedom systems respond in independent motions of the individual modes. Then the time responses of the equivalent systems, taken in various proportion according to appropriate modal participation factors, are combined to yield the complete time response of the actual multi-story structure.

The modal superposition principle may be expressed in mathematical form. For a multi-story structure idealized as n discrete masses, Fig. 8, it may be shown (9) that, corresponding to equation (3) of the one-mass system, the lateral displacement of the i^{th} mass, m_i, relative to the ground is

$$u_i(t) = - \sum_{r=1}^{n} \frac{T_r}{2\pi} A_i^{(r)} \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \int_0^t \ddot{x}(\tau) e^{-\frac{2\pi}{T_r} n_r (t-\tau)} \sin \frac{2\pi}{T_r}(t-\tau) d\tau \tag{10}$$

which represents the contribution of all the normal modes. The r superscript is used to identify the particular mode of vibration.

T_r = the period of the r^{th} normal mode of vibration
$A_i^{(r)}$ = the relative displacement of the i^{th} mass in the r^{th} mode.

The A's describe the shapes of the principal modes of vibration and for each modal period there is a set of these coefficients; since only the shape of a normal mode is significant, it is the ratios of the A's which are to be determined and not the values of the individual A's. It is generally the practice to assign one of the amplitudes to be unity, in which case the modes are said to be normalized to unity.

Note that the earthquake response problem is again controlled by the characteristics of the ground disturbance, as expressed by the integral term, and the characteristics of the structure. The structural characteristics are defined by the modal shapes and periods, which are functions of the mass and stiffness distribution of the structure, and the modal damping values. The mode shapes and natural periods of a structure may be evaluated by numerical procedures (4) or through the frequency determinant approach (8). General computational procedures are not available for evaluating the damping of a structure and the value of this parameter must be based on judgment and the result of experiments.

The contribution of the r^{th} mode alone is

$$u_i^{(r)}(t) = -\frac{T_r}{2\pi} A_i^{(r)} \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)2}} \int_0^t \ddot{x}(\tau)\, e^{\frac{-2\pi n_r (t-\tau)}{T_r}} \sin\frac{2\pi}{T_r}(t-\tau)\, d\tau \qquad (11)$$

In design it is the maximum response which is of primary interest, rather than the complete time response. Since the maximum value of the integral in equation (11) is simply the velocity spectrum in the r^{th} mode, it follows that the maximum response in this mode is

$$[u_i^{(r)}]_{max} = -\frac{T_r}{2\pi} A_i^{(r)} A_r S_v^{(r)} = A_i^{(r)} \alpha_r S_d^{(r)} \qquad (12)$$

where α_r = modal participation factor of the r^{th} mode.

$$\alpha_r = \frac{\sum_{j=1}^{n} m_j A_j^{(r)}}{\sum_{j=1}^{n} m_j A_j^{(r)^2}} \qquad (13)$$

Other quantities of interest can be similarly expressed. By differentiating equation (10), the absolute acceleration of any mass, $\ddot{y}_i$; can be obtained since $\ddot{y}=\ddot{x} + \ddot{u}$. In the r^{th} mode this can be written as

$$[\ddot{y}^{(r)}]_{max} = \frac{2\pi}{T_r} A_i^{(r)} S_v^{(r)} \alpha_r = A_i^{(r)} \alpha_r S_a^{(r)} \qquad (14)$$

The effective lateral seismic force generated on the i^{th} mass follows directly since

$$[F_i^{(r)}]_{max} = m_i \ddot{y}_{i\,max}^{(r)} = m_i \frac{2\pi}{T_r} A_i^{(r)} S_v^{(r)} \alpha_r = m_i A_i^{(r)} \alpha_r S_a^{(r)} \qquad (15)$$

For future reference it is convenient to express this latter result in a form frequently embodied in design codes and directly comparable to the results of our analysis of a one-mass system. This involves the determination of the r^{th} mode base shear and its resolution into lateral forces applied to the structure. In any mode, to satsify dynamic equilibrium, the sum of all the effective forces acting on the structure must equal the base shear or

$$V_B^{(r)} = \sum_{i=1}^{n} F_i^{(r)} = \frac{2\pi}{T_r} \alpha_r S_v^{(r)} \sum_{i=1}^{n} m_i A_i^{(r)} \tag{16}$$

After substituting the values of α_r, equation (13), this may be expressed as

$$V_B^{(r)} = \frac{2\pi}{T_r} S_v^{(r)} \frac{\left(\sum_{j=1}^{n} m_j A_j^{(r)}\right)^2}{\left(\sum_{j=1}^{n} m_j A_j^{(r)2}\right)} = \left(\frac{2\pi}{T_r} \frac{S_v^{(r)}}{g}\right) W^{(r)} \tag{17}$$

which is directly equivalent to equation (9) for the one-mass system. $W^{(r)}$, which may be considered as the effective weight of the equivalent one-mass system for the r^{th} mode, is given by

$$W^{(r)} = \frac{\left(\sum_{j=1}^{n} w_j A_j^{(r)}\right)^2}{\left(\sum_{j=1}^{n} w_j A_j^{(r)2}\right)} \tag{18}$$

where w_j represents the weight of the mass concentrated at the j^{th} floor.

The resolution of the base shear into equivalent lateral seismic forces distributed throughout the height of the structure is found after α_r is eliminated from equations (15) and (16). Thus

$$F_i^{(r)} = \frac{V_B^{(r)} \left(w_i \; A_i^{(r)}\right)}{\left(\sum_{i=1}^{n} w_i A_i^{(r)}\right)} \tag{19}$$

Earthquake Spectrum Analysis for Multi-Story Structures

The above analysis reveals that the concept of response spectrum may now be combined with that of modal superposition to yield an approximate solution for the maximum response of a multi-story structure. The procedure for calculating the earthquake generated motions and resulting forces may be summarized in four separate steps

(1) Calculate the modal periods and associated normal mode shapes of the structure. Assign appropriate values of damping to each normal mode.

(2) For each mode, read directly from the response spectrum appropriate to the particular ground disturbance being considered, the maximum modal response for the appropriate period and damping established in step (1).

(3) Evaluate the modal participation factors, equation (13), from the data of step (1) and obtain the contribution of each mode to the total response of the complete system by multiplying the appropriate spectrum quantity by the modal participation factor and the mode shape.

(4) Combine the values of the individual mode contributions from step (3) to find the total response. The most suitable method of combination of modal responses is discussed below.

Combination of Modal Responses

Since the individual modal maxima will normally occur at different times, Fig 2, the sum of the absolute values of each modal contribution gives an upper bound to the total system response which, in general, would be overly conservative. The error arising from an absolute superposition of the spectral maxima can be overcome, in part, by taking the total maximum response as equal to the square root of the sum of the squares of the individual modal maxima. This criterion, which is based on statistical considerations (10, 11), yields the maximum probable earthquake response and evidently leads to lower values than that given by the absolute sum.

Since the bulk of the energy of vibration of a multi-story structure is normally contained in the lower modes of vibration, it is generally sufficient to consider only the first 3 or 4 modes in the modal combination (12).

Illustrative Example

To illustrate modal spectrum analysis we will apply the preceding principles to evaluate the dynamic response of a 3-story structure when subjected to the El Centro earthquake of 1940. The structure is idealized as shown in Fig. 9 by assuming its mass concentrated at the floor levels. The floor diaphragms are assumed to be infinitely rigid so that there are no rotations at the points of mass concentration.

For the given masses and stiffnesses the natural periods and mode shapes were calculated by an automatic digital computer using the frequency determinant approach with the results shown in Fig. 9. The spectrum responses of the one-degree-of-freedom systems equivalent to these normal modes were next read directly from Fig. 4. These depend only on the mode periods and damping. In this example the spectrum values selected are the displacement responses; the values of S_d appropriate to each mode are also tabulated in Fig. 9.

The modal participation factors are evaluated by applying equation (13) to the information shown in Fig. 9. The product of the participation factors, mode shapes and appropriate spectrum quantities, either S_a, S_v or S_d, leads to the response functions desired and tabulated in Table I, which presents modal values of displacements, accelerations, effective seismic forces and inter-story shears. The modal displacements, accelérations and forces are obtained from equations (12), (14), and (15) respectively. In each mode, the inter-story shear is found by an algebraic summation of the effective forces from the top downward.

The maximum possible response values, column 6, are found by summation of the absolute individual mode values. In computing the combined response, signs are not attached to the individual mode values since each mode can operate in either direction, Fig. 2. A root mean square calculation is performed with the individual modal values of displacements, accelerations and shears to yield the maximum probable values of these quantities, column 7. Note that the first mode makes the major contribution to the shears; the higher modes are relatively more important in the case of accelerations.

For tall structures exhibiting many degrees-of-freedom, the computational problem involved in an analysis of the above type is extremely laborious and is best done by computing machines.

Influence of Inelastic Behaviour and Other Factors

The preceding account of structural behaviour is based on a purely elastic response. Structures designed in accordance with existing seismic code provisions do not generally show evidence of the distress which might be expected if the dynamic lateral forces calculated by elastic analysis, which exceed signigicantly the code design forces, were actually generated as a result of earthquake motions to which they have been subjected. It is widely accepted that this discrepancy may be allowed for, at least in part, by acceptance of some plastic deformations of buildings or their foundations, during severe earthquakes. Analysis of inelastic structural response indicates that moderate plastic deformations absorb large amounts of energy from earthquake generated motions. This has the effect of reducing the response of the system and limiting the lateral forces developed in the structure. This will be discussed in the following lecture.

While the theoretical principles outlined above focus attention on important parameters influencing the behaviour of structures during earthquakes, it should be recognized that in addition to inelastic deformations, other factors may play a significant role in determining dynamic structural response. Among these are: soil conditions at the site, building and ground interaction, and alterations to earthquake motions due to interference from the structure itself (feedback). Many of these problems are complex and have not been completely investigated or are not yet fully understood; some will be discussed in later lectures.

Summary

1. The seismic forces generated in a structure depend on the characteristics of the earthquake ground motions and the characteristics of the structure.

2. The response spectrum provides a way of separating that part of the response calculation which depends upon the earthquake disturbance from that part which involves mainly the structural properties. From the spectrum curves it is possible to read off the spectrum response of a single-degree-of-freedom system; the maximum response of a mode of vibration of a more complex system can also be determined from the response spectrum.

3. The normal mode shapes, natural periods and dampings of a building are the characteristics of the structure which control its response to any disturbance. The mode shapes and periods are functions of the mass and stiffness distribution of the structure.

4. Small amounts of damping produce large reductions in the maximum response. The damping is a function of the material and type of construction of a structure.

5. The maximum response of multi-story structures can be evaluated by modal superposition principles. The spectrum concept provides a convenient approach to modal superposition.

6. The degree to which each normal mode contributes to the total response is given by the product of the modal participation factor, the modal shape and the appropriate modal response spectrum. This involves a prior calculation for the determination of the modal periods and shapes.

7. A root mean square combination of the independent modal responses provides the most suitable means of establishing the maximum probable total response developed in the structure. It is generally satisfactory to include only the first three or four mode contributions in this calculation.

8. The ability of a structure to deform plastically has the effect of limiting the lateral forces generated by an earthquake. As a result, the strength and stiffness requirements can be less than those demanded by a strictly elastic analysis. Other factors, such as soil conditions and building and ground interaction, may play an important role in determining dynamic structural response.

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TABLE I MODAL SPECTRUM RESPONSES

(1) Response Quantity	(2) Mass	(3) Mode r = 1	(4) Mode r = 2	(5) Mode r = 3	(6) Max. Possible Response	(7) Max. Probable Response
$u_i^{(r)}$ = mass displacement (in.)	i = 3	3.570	-0.449	0.028	4.047*	3.595**
$= A_i^{(r)}\,\alpha_r\,S_d^{(r)}$	2	2.220	0.280	-0.081	2.581	2.265
	1	0.896	0.202	0.144	1.242	0.938
$\ddot{y}_i^{(r)}$ = mass acceleration (ft./sec.^{2})	i = 3	20.70	-11.16	1.586	33.45	23.50
$= A_i^{(r)}\,\alpha_r\,S_d\,\omega_r^2$	2	12.88	6.94	-4.795	24.62	15.40
	1	5.17	5.00	8.530	18.70	11.10
$F_i^{(r)}$ = effective forces (kips)	i = 3	96.31	-51.92	7.79		
$= A_i^{(r)}\,\alpha_r\,S_d\,m_i\,\omega_r^2$	2	119.64	64.79	-44.65		
	1	48.30	46.56	79.50		
Shears (kips) 3rd story	i = 3	96.31	-51.92	7.79	156.02	109.69
2nd story	2	215.95	12.87	-36.85	265.67	219.45
Base	1	264.25	59.44	42.65	366.34	274.19

$$\omega_r = \frac{2\pi}{T_r}$$

* $3.570 + 0.449 + 0.028 = 4.047$

** $\sqrt{3.570^2 + 0.449^2 + 0.028^2} = 3.595$

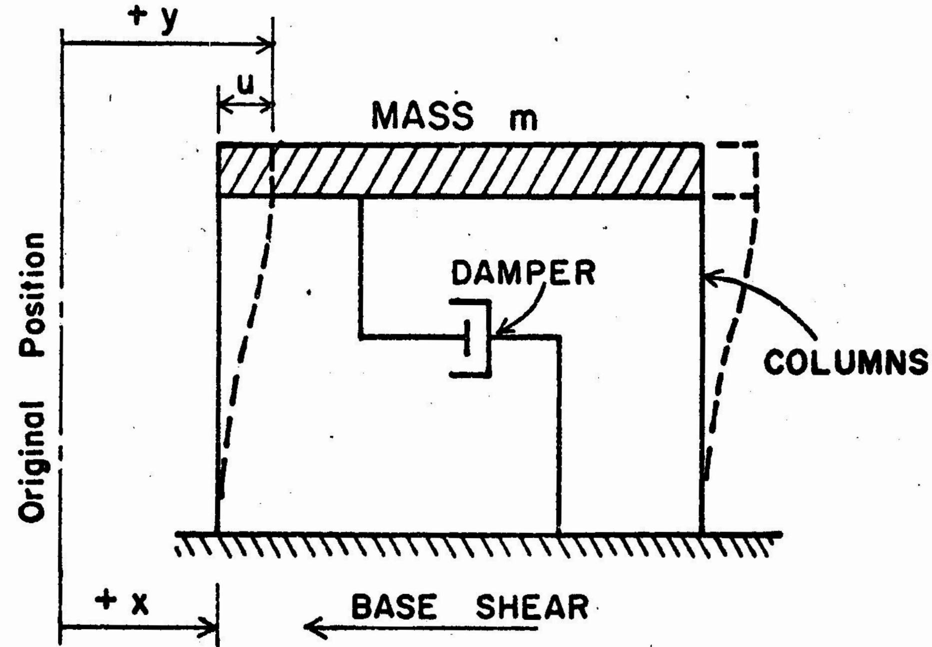


FIG. 1 SCHEMATIC REPRESENTATION OF ONE-STOREY FLEXIBLE STRUCTURE

$$V = k\,u \tag{1}$$

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{x} \tag{2}$$

$$u(t) = \underbrace{\frac{T}{2\pi}}_{\text{characteristic of structure}} \underbrace{\int_0^t \ddot{x}(\tau)\, e^{-n\left(\frac{2\pi}{T}\right)(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\, d\tau}_{S_t \text{ characteristic of earthquake}} \tag{3}$$

$S_v = (S_t)_{max} =$ max. relative velocity

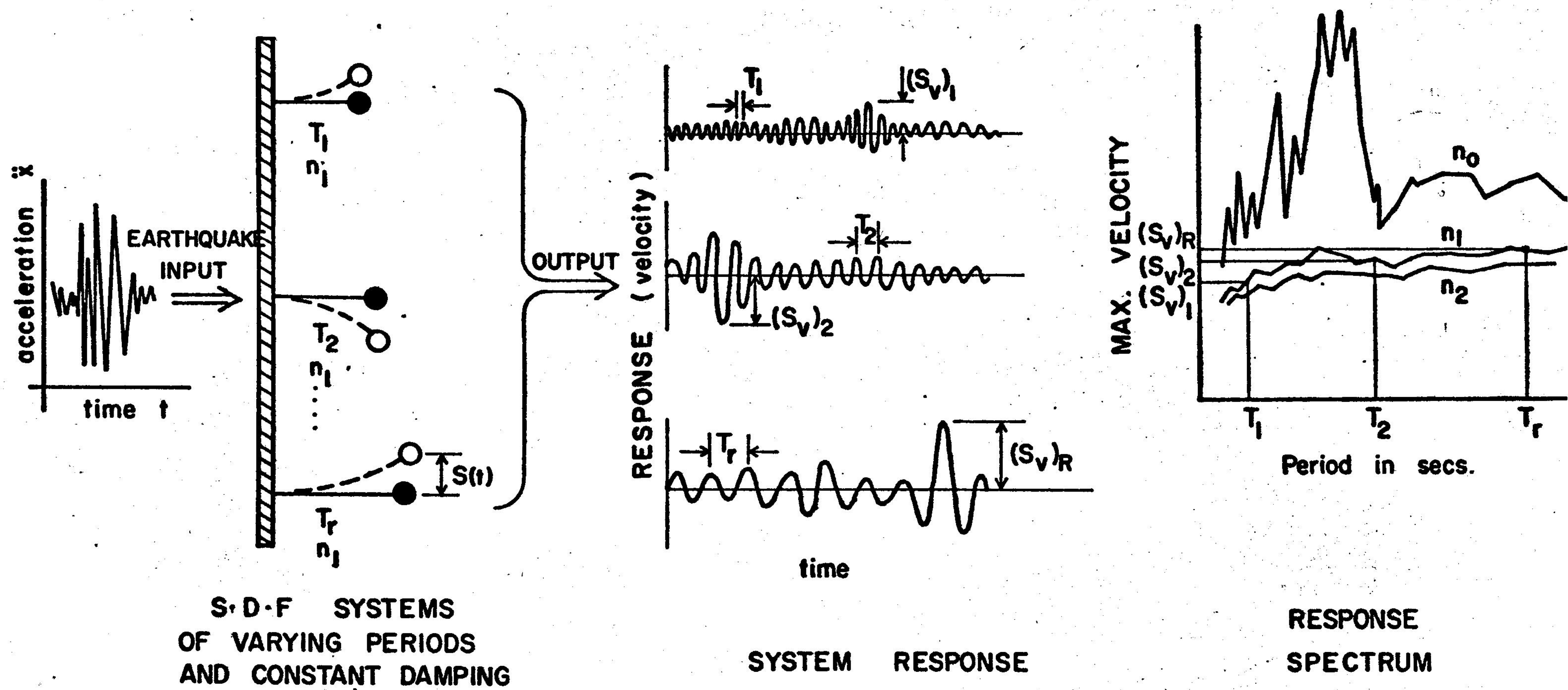


FIG. 2 SCHEMATIC INTERPRETATION OF RESPONSE SPECTRUM

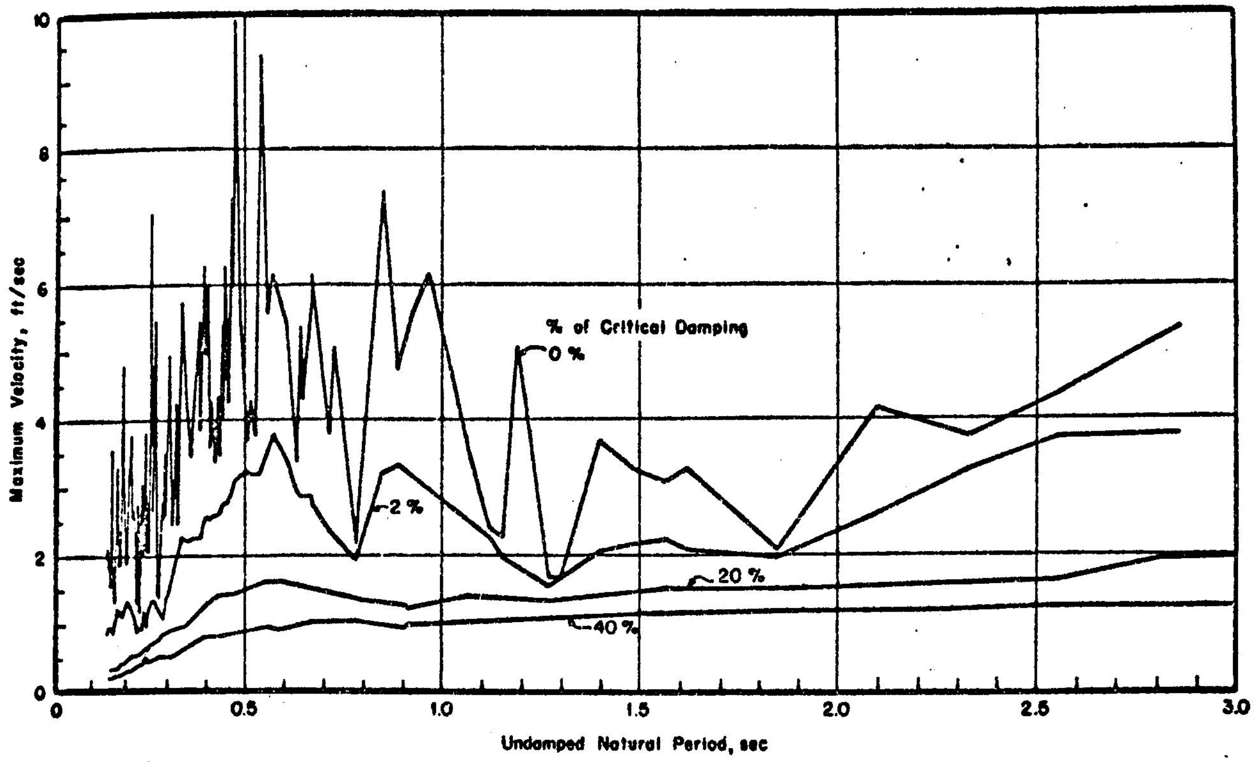


Fig. 3 —Velocity spectrum for El Centro, Calif., earthquake, May 18, 1940. Component N–S.
(from Ref. 7)

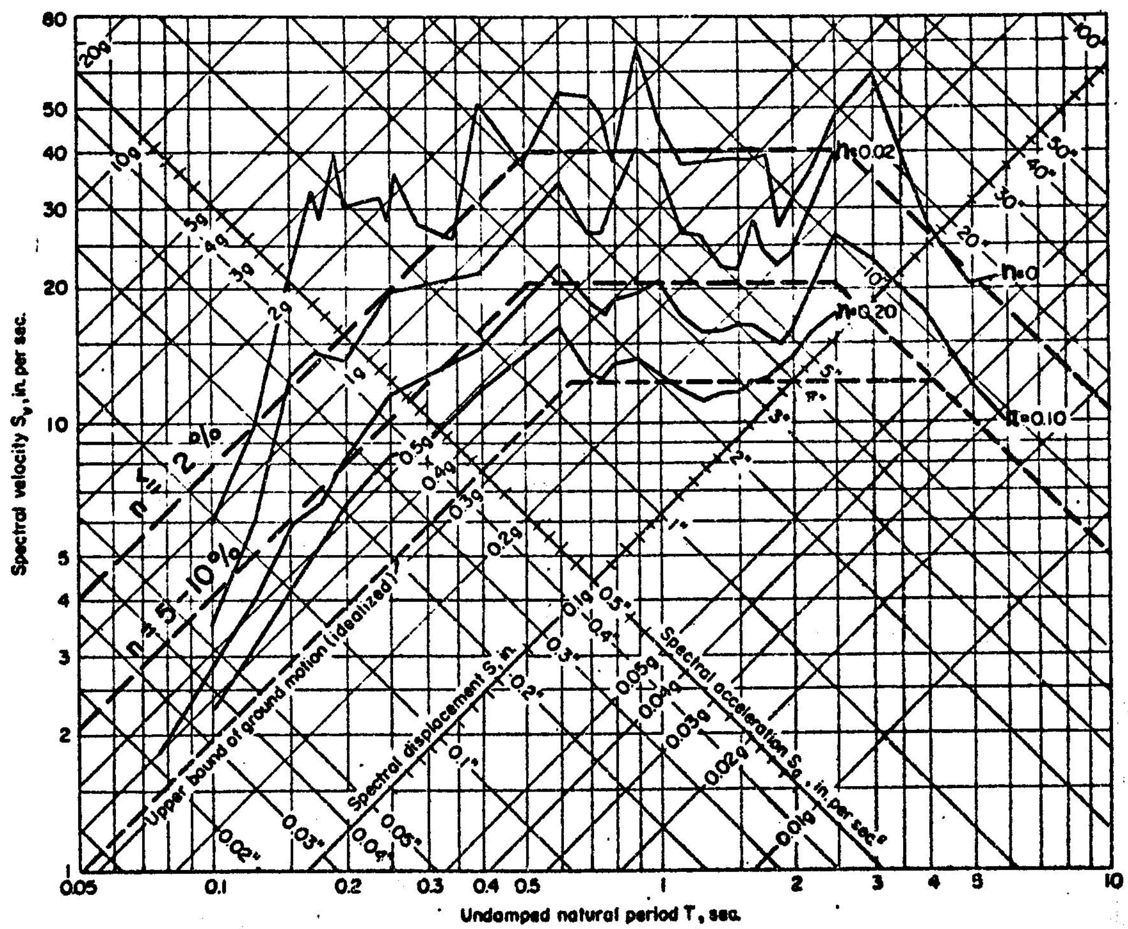


Fig. 4 Response spectra for elastic systems, 1940 El Centro earthquake.
(from Ref. 4)

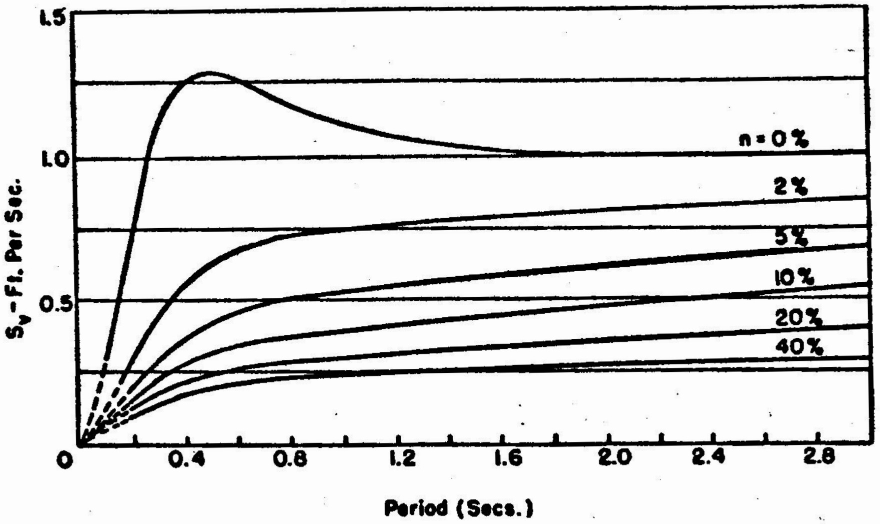


FIG. 5 - AVERAGE VELOCITY SPECTRUM CURVES
(from Ref. 1)

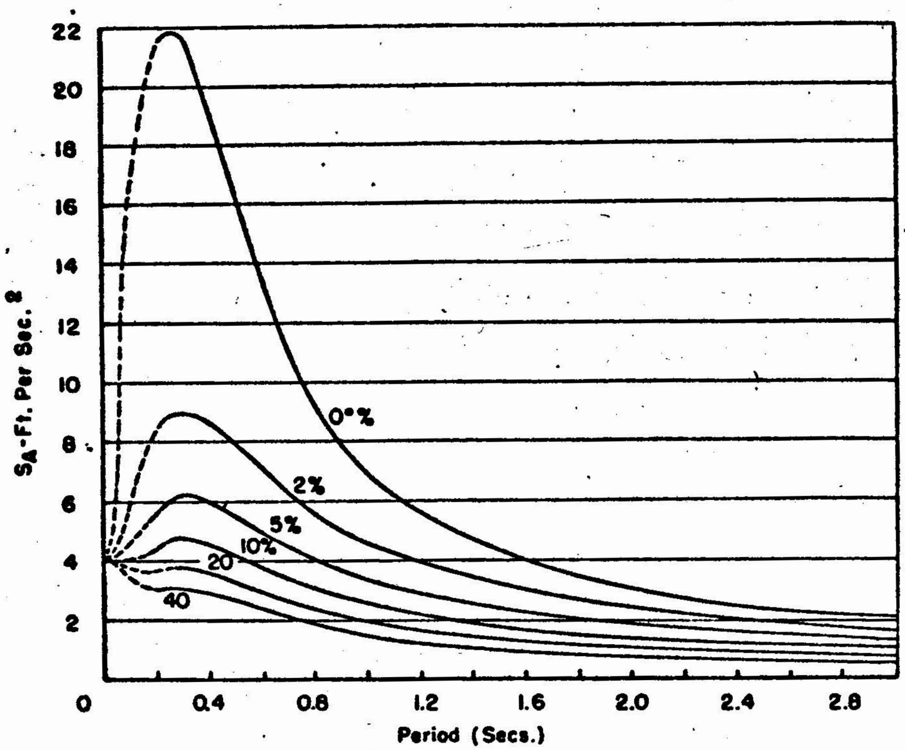


FIG. 6 - AVERAGE ACCELERATION SPECTRUM CURVES
(from Ref. 1)

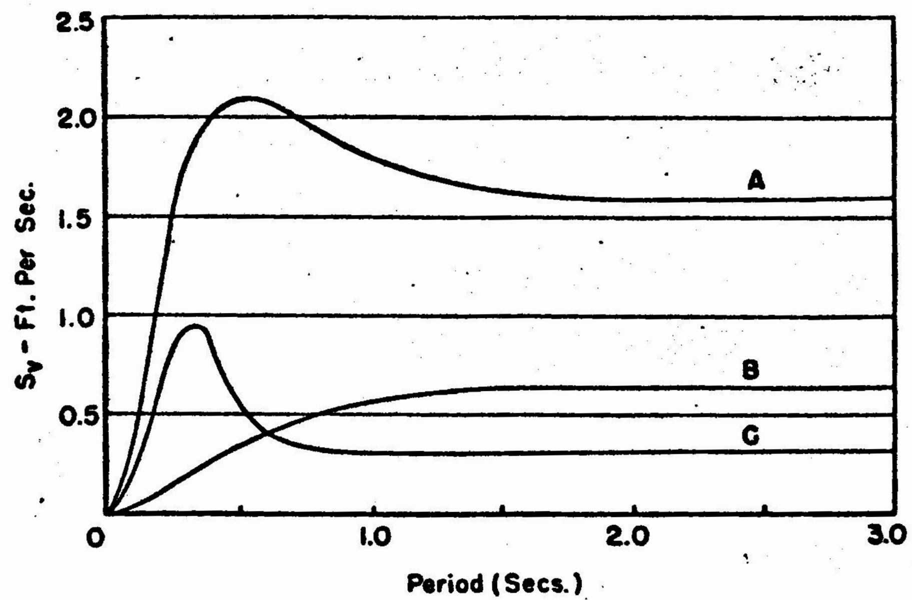
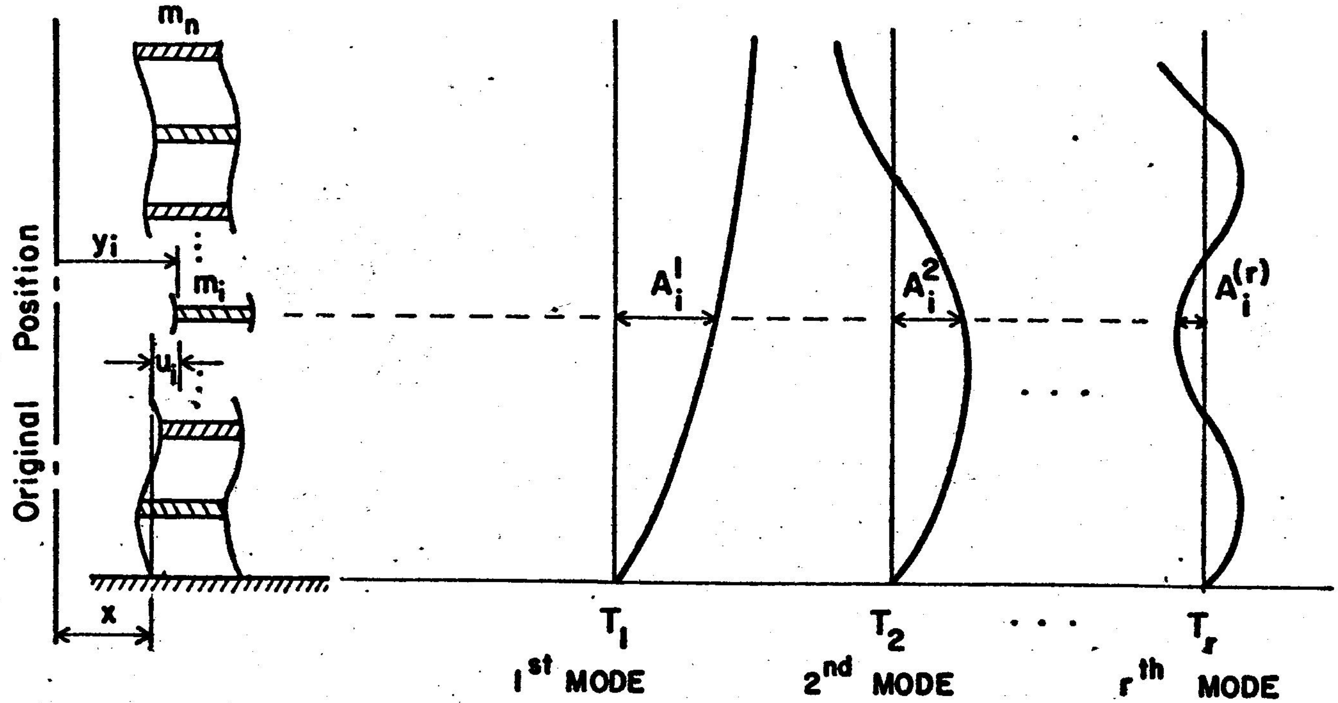


FIG. 7 - UNDAMPED VELOCITY SPECTRA
(from Ref. 1)

A. ± 25 Miles From Center Of Large Shock

B. ± 70 Miles From Center Of Large Shock

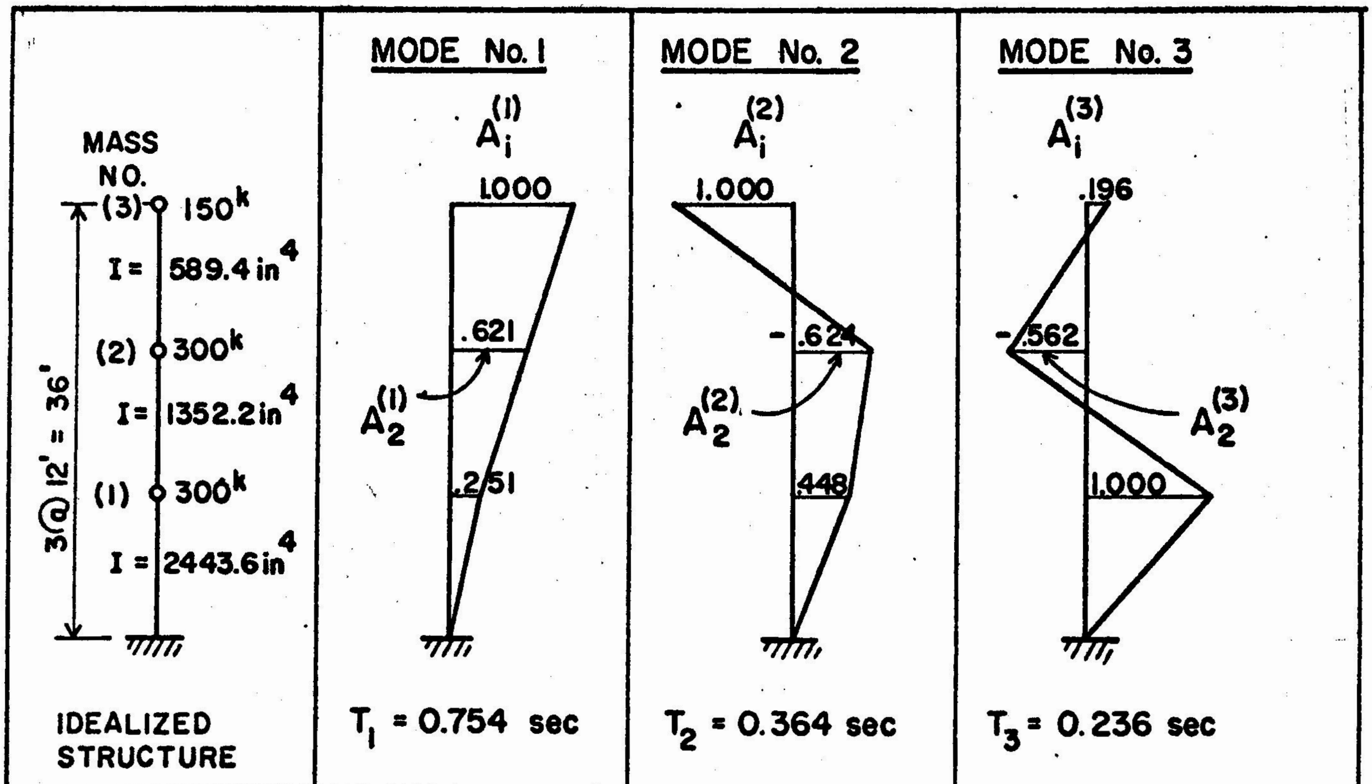
C. ± 10 Miles From Center Of Small Shock



(a) IDEALIZED STRUCTURE

(b) NORMAL MODEL SHAPES

FIG. 8 MULTI-STORY STRUCTURE EXCITED BY HORIZONTAL GROUND MOTION



	MODE No. 1	MODE No. 2	MODE No. 3
DATA	n_1 = 5-10%	n_2 = 5-10%	n_3 = 5-10%
	a_1^* = 1.446	a_2 = -0.525	a_3 = 0.402
	$S_d^{(1)}$ = 2.473 in	$S_d^{(2)}$ = 0.857 in	$S_d^{(3)}$ = 0.360 in

$$^*a_1 = \frac{\sum_{j=1}^{3} m_j A_j^{(1)}}{\sum_{j=1}^{3} m_j A_j^{(1)2}} = \frac{(150\times 1 + 300\times .621 + 300\times .251)}{150\times 1^2 + 300\times .621^2 + 300\times .251^2} = 1.446$$

FIG. 9 PARAMETERS AND MODE SHAPES FOR ILLUSTRATIVE EXAMPLE